Applying Bayesian data analysis to learn about periodic modulations in pulsars

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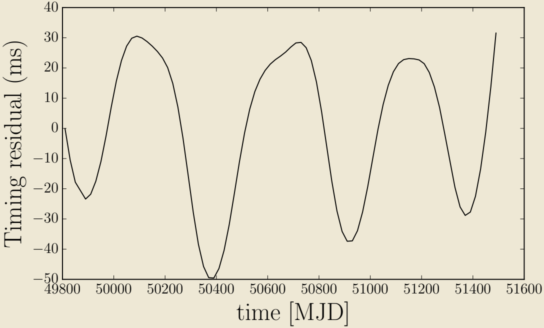
Motivation

- Pulsars allow us to study the composition and structure of neutron stars
- Variations in the arrival time of pulsations, often termed timing-noise, tell us that we have unmodelled physics
- There is a lot of variation in the observed timing-noise, but a few show highly periodic variations $\sim 1 - 10$ yrs

- Today I will discuss two models able to explain these periods

Periodic modulations: B1828-11

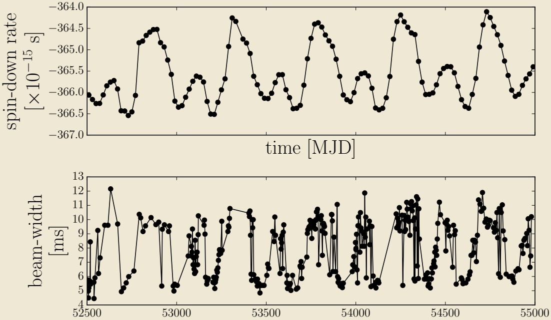
- Demonstrates periodic modulations at 500 days (with harmonics)
- Periodicity observed in: timing-residual, beam-width, frequency, and spin-down rate



: Data courtesy of Lyne at al. (2010)

- One explanation for these modulation is *precession*

Precession explains the smoothly modulated double-peaked spin-down

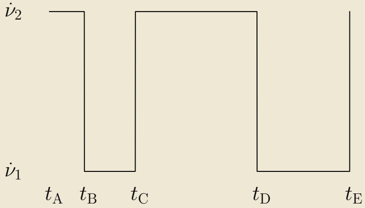


: Data courtesy of Lyne at al. (2010)

However, the beam-width appears to suddenly switch between two states

Model: Switching

- Lyne et al. (2010): the magnetosphere undergoes sudden periodic switching between two states
- The smooth modulation in the spin-down is due to time-averaging of this underlying spin-down model
- To explain the *double-peak*, Perera (2014) suggested four times were required
- Lyne (2010) reject precession in favour of ‘switching’ model



There are arguments for and against both the *precession* and *switching* models, but is there enough evidence to rule either out?

We would like to quantify how well the two models fit the data. To do this we will use Bayes theorem:

$$P(\mathcal{M}_j|\mathbf{y}_{\rm obs}) = P(\mathbf{y}_{\rm obs}|\mathcal{M}_j)\frac{P(\mathcal{M}_j)}{P(\mathbf{y}_{\rm obs})}. \tag{1}$$

The odds ratio:

$$\mathcal{O} = \frac{P(\mathcal{M}_A|\mathbf{y}_{\rm obs})}{P(\mathcal{M}_B|\mathbf{y}_{\rm obs})} = \frac{P(\mathbf{y}_{\rm obs}|\mathcal{M}_A)}{P(\mathbf{y}_{\rm obs}|\mathcal{M}_B)}\frac{P(\mathcal{M}_A)}{P(\mathcal{M}_B)}. \tag{2}$$

If we have no preference for one model or the other then set

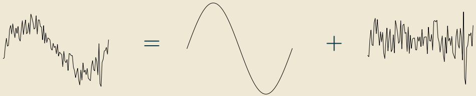
$$\frac{P(\mathcal{M}_A)}{P(\mathcal{M}_B)} = 1. \tag{3}$$

and ‘let the *data* speak for itself’

Bayesian data analysis: Model fitting

For a signal in noise:

$$y^{\text{obs}}(t_i|\mathcal{M}_j,\vec{\theta}) = f(t_i|\mathcal{M}_j,\vec{\theta}) + n(t_i) \tag{4}$$



If the noise is stationary and can be described by a normal distribution:

$$y^{\text{obs}}(t_i|\mathcal{M}_j,\vec{\theta}) - f(t_i|\mathcal{M}_j,\vec{\theta}) \sim N(0,\sigma) \tag{5}$$

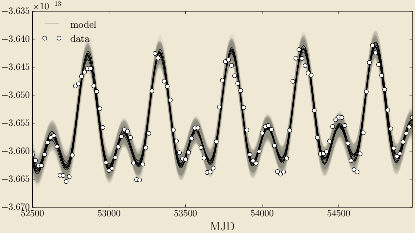
Then the likelihood is:

$$P(y_i^{\text{obs}}|\mathcal{M}_j,\vec{\theta},\sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{ \frac{-\left(f(t_i|\mathcal{M}_j,\vec{\theta}) - y_i\right)^2}{2\sigma^2} \right\} \tag{6}$$

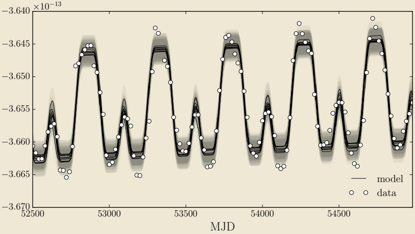
We use Markov chain Monte Carlo (MCMC) methods to ‘fit’ the model to the data. First we check that we have a good fit, then calculate the odds-ratio.

Bayesian data analysis: Checking the fit

Precession model:



Switching model:



Results

$$\frac{P(\mathcal{M}_{Precession}|\mathbf{y}_{\mathrm{obs}})}{P(\mathcal{M}_{Switching}|\mathbf{y}_{\mathrm{obs}})} \approx 1$$

Conclusions

- We should use data analysis tools to quantify our belief in models
- For the spin-down alone, there is no evidence to suggest switching is preferable to precession
- We now combine the spin-down and beam width date to fully resolve the question
- In the future, we intend to form a hybrid model where the precession biases the switching