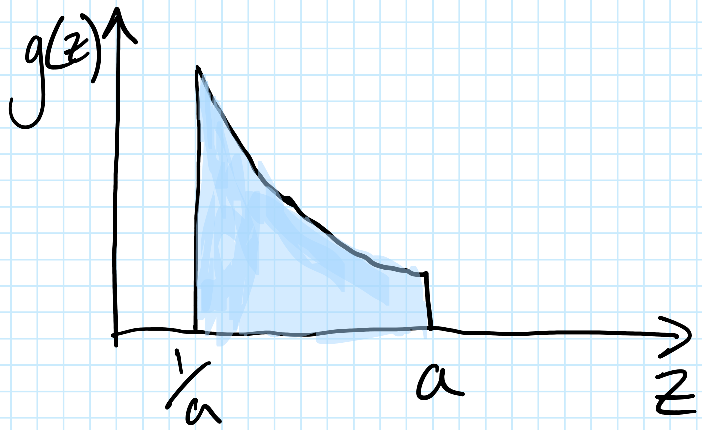
Stretch Move

Thursday, 11 March 2021 11:11

Drawing samples from the Goodman & Weare (2010) distribution:

$$g(z) = \begin{cases} z^{-1/2} & \text{if } z \in \left[\frac{1}{a}, a\right] \\ 0 & \text{otherwise} \end{cases}$$



First, the normalisation:

$$\int_{1/a}^{a} C\, z^{-1/2}\, dz = 1$$

$$\Rightarrow 2\sqrt{z}\,\Big|_{1/a}^{a} = \frac{1}{C}$$

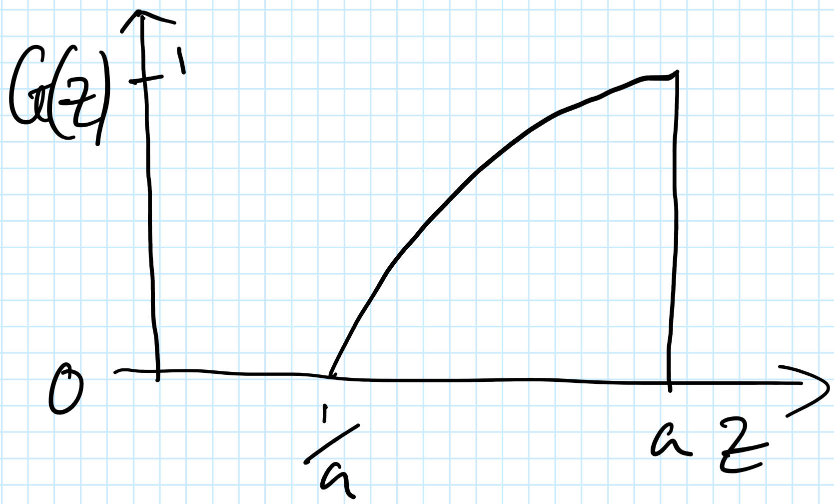
$$\Rightarrow C = \left(2\sqrt{a} - 2\sqrt{\frac{1}{a}}\right)^{-1} = \frac{\sqrt{a}}{2(a-1)}$$

So $g(z) = C \frac{1}{\sqrt{z}}$ between $\frac{1}{a}$, a

$$\Rightarrow G(z) = \int_{1/a}^{z} g(z')\,dz'$$

$$= C\left[2\sqrt{z} - 2\sqrt{\tfrac{1}{a}}\right]$$

Sketching the CDF



To draw from Z invert the CDF

$$u = C\left[2\sqrt{z} - 2\sqrt{\tfrac{1}{a}}\right]$$

$$= \frac{\sqrt{a}}{2(a-1)}\left[2\sqrt{z} - 2\sqrt{\frac{1}{a}}\right]$$

$$= \frac{\sqrt{za}}{(a-1)} - \frac{1}{a-1}$$

$$\Rightarrow \quad u(a-1) = \sqrt{za} - 1$$

$$\sqrt{za} = u(a-1) + 1$$

$$z = \frac{\left(u(a-1)+1\right)^2}{a}$$

Let $u \sim U(0,1) \Rightarrow z$ are draws from $g(z)$.